Prospects for Low Cost Fusion Development

Contact: Gordon Long — glong@mitre.org

November 2018

JSR-18-011

JASON
The MITRE Corporation
7515 Colshire Drive
McLean, Virginia 22102-7508
(703) 983-6997

3 PHYSICS BACKGROUND

The energy contained in atomic nuclei can be tapped either by splitting a heavy nucleus into smaller ones (fission) or combining two small nuclei into a larger one (fusion). Fission (aka nuclear power) is a well-developed energy technology supplying 11% of the world's electricity[40], as well as propulsion for submarines and naval surface ships. Fusion, although common in the cosmos (it powers the stars), has so far found meaningful terrestrial application only in nuclear weapons. Achieving sustained and controlled fusion reactions would open the door to a new source of energy with intriguing advantages relative to existing sources.

3.1 Cross Sections

If two nuclei are to fuse, they must be brought to a separation r ~ several femtometers (1 fm = 10^{-15} meter), the characteristic size of light nuclei. The energy required to do that (the Coulomb barrier, U) is given by

$$U = \frac{Z_1 Z_2 e^2}{r} = 450\ \mathrm{keV}\ Z_1 Z_2 \left(\frac{r}{3\ \mathrm{fm}}\right)^{-1}$$

where $Z_{1,2}$ are the atomic numbers of the fusion nuclei. While the coulomb barrier is easily surmounted by an accelerator beam, energy applications involve thermonuclear fusion, where the nuclei collide due to their thermal motion in a hot plasma. Here the energies are necessarily much lower (a temperature of 1.2×10^7 K is only 1 keV), so that if the nuclei are to fuse, they must tunnel quantum mechanically thorough the Coulomb barrier (Fig. 5). These considerations imply that lighter nuclei (a more quantal system that tunnels more readily), smaller atomic numbers (lower coulomb barrier), and higher temperatures (more energetic collisions) will optimize the rate of thermonuclear fusion.

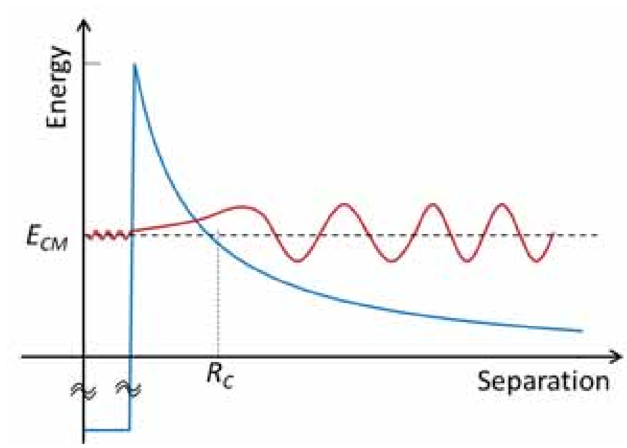

Figure 5. When nuclei collide and reach the classical turning point (R_C), there is a finite probability that the relative motion will tunnel through the Coulomb barrier and fuse.

[40] http://www.world-nuclear.org/information-library/current-and-future-generation/nuclear-power-in-the-world-today.aspx

Table 2. Energetics of low-Z reactions considered for fusion energy in MeV.

D + D	$\xrightarrow{50\%}$	T(1.011) + p(3.022)
D + D	$\xrightarrow{50\%}$	^{3}He(0.820) + n(2.449)
p + T	$\longrightarrow$	^{3}He + n − 0.764
D + T	$\longrightarrow$	^{4}He (3.561) + n(14.029)
T + T	$\longrightarrow$	^{4}He + 2n + 11.332
D + ^{3}He	$\longrightarrow$	^{4}He(3.712) + p(14.641)
T + ^{3}He	$\xrightarrow{59\%}$	^{4}He + n + p + 12.096
T + ^{3}He	$\xrightarrow{41\%}$	^{4}He(4.800) + D(9.520)
^{3}He + ^{3}He	$\longrightarrow$	^{4}He + 2p + 12.860

There are several possibilities for reactions among the isotopes of hydrogen and helium.[41] In Table 2, *p* and *n* are proton and neutron, respectively, D = ^{2}H (deuteron), and T = ^{3}H (triton). The numbers represent energy in MeV. The highlighted D + T reaction is favored in energy production schemes due to its large cross section (see below). There are other, minor reaction branches among these reactants, for example D + D → ^{4}He + ϒ (24 MeV), which proceeds at a rate some 10^{7} times smaller than the branches shown. The reaction p + p → D + e^{+} + ν_e, which is the dominant process in the Sun, has far too low a rate to be practical on Earth.

The DT reaction has several drawbacks. One is that 80% of the energy produced is carried by a 14.4 MeV neutron, which cannot be confined by a magnetic field, is highly penetrating (a 5 cm range in steel), and induces long-lived radioactivity as well as embrittlement in surrounding materials (by comparison, fission neutrons have energies around 2 MeV). Replacement of reactor walls will increase operating costs, cause down-time, and require handling low-level radioactive waste. A second drawback of the DT reaction is that tritium is radioactive, undergoing a beta decay to ^{3}He with a half-life of 12.3 years. As a result, the tritium fuel is not found in nature (unlike deuterium, which is 0.016% of natural hydrogen), but must be continuously produced ("bred") during reactor operation, usually by the ^{6}Li + n → ^{4}He + T reaction.

The multiple problems engendered by neutrons have led to consideration of aneutronic reactions such as ^{3}He + ^{3}He → ^{4}He + 2p + 12.86 MeV, and ^{11}B + p → 3 ^{4}He + 8.7 MeV. While the first suffers from both a low reaction rate (only 10^{-3} that of DT at a temperature of 100 keV) and a relative rarity of ^{3}He in nature (only 0.000137% of terrestrial Helium), some have considered ^{11}B + p a promising candidate (^{11}B is 80.1% of natural Boron), though the kinetics of this reaction are less favorable than the alternatives (see below).

Nuclear reaction rates are characterized by the cross section, σ, which expresses the probability of reaction in terms of an equivalent target area presented to the approaching particle. The cross section

[41] http://www.kayelaby.npl.co.uk/atomic_and_nuclear_physics/4_7/4_7_4.html

depends upon the speed of the collision (e.g., collisions at a higher energy will more easily tunnel through the Coulomb barrier); it is conventionally measured in units of barns (1 barn = 10^{-24} cm^2). Cross sections for reactions at thermonuclear energies are dominated by the quantum penetration of the Coulomb barrier, and so have the form

$$\sigma = \frac{S(E)}{E} e^{-R/E^{1/2}}$$

where E is the center-of-mass energy of the collision, $S(E)$ is a nuclear structure factor which in most cases varies slowly with energy (for a non-resonant reaction), and

$$R = Z_1 Z_2 \pi e^2 \sqrt{\frac{2m}{\hbar^2}} = 31.39 \text{ keV}^{1/2} Z_1 Z_2 \left(\frac{m}{1 \text{ AMU}}\right)^{1/2},$$

where $m = m_1 m_2/(m_1 + m_2)$ is the reduced mass of the collision. R characterizes the strength of the Coulomb barrier in the reaction.

Figure 6 gives energy-dependent cross sections for reactions of interest. Table 3 shows approximate values of S and R for some common reactions, neglecting the energy dependence of $S(E)$.[42] In some reactions, such $p^{11}B$, sharp resonances in $S(E)$ contribute substantially to the cross section. Energy-dependent interpolation formulas for $S(E)$ for common reactions are in Ref. 43. DT has the largest cross section at thermonuclear energies both because of the low atomic numbers involved and because of a resonance in the compound nucleus ^5He.

Table 3: Cross section parameters for some fusion reactions

Reaction	*S* (barns-keV)	*R* ($keV^{1/2}$)
$D–D_p$	52.6	31.39
$D–D_n$	52.6	31.39
D–T	9821	34.37
T–T	175	38.41
$D–^3He$	5666	68.74
$T–^3He$	2422	76.82
$^3He–^3He$	5500	153.7

[42] E. G. Adelberger, et al. "Solar fusion cross sections." *Reviews of Modern Physics* **70**, 1265 (1998).

[43] H.-S. Bosch and G. M. Hale. "Improved formulas for fusion cross-sections and thermal reactivities." *Nuclear Fusion* **32**, 611 (1992)

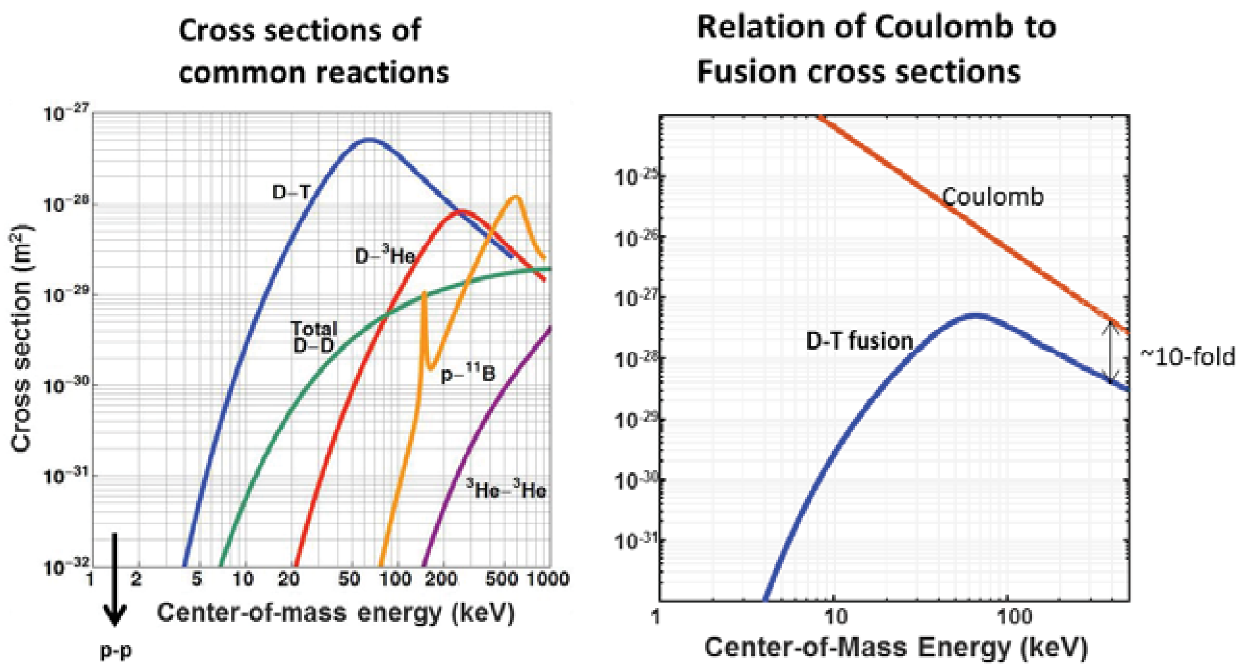

Figure 6. Fusion cross sections. A) Cross sections of common fusion reactions as a function of center-of-mass kinetic energy. The p-p reaction, which dominates in the Sun, has too low a cross section to be useful on Earth, due to the participation of the Weak interaction. B) Comparison of Coulomb and fusion cross sections for the D-T reaction. At all energies, Coulomb scattering is more likely than fusion. D-T data from [Bosch, H-S., and G. M. Hale. "Improved formulas for fusion cross-sections and thermal reactivities." Nuclear fusion 32.4 (1992): 611.]. The parameter $\ln \Lambda$ was set to 10.

Fusion reactions compete with non-productive Coulomb scattering, which can cause loss of particles and energy from the plasma. The Coulomb scattering cross-section decreases as kinetic energy increases because the classical Coulomb turning radius scales as $1/E$. Integrating over the differential cross-section yields an effective Coulomb cross-section:

$$\sigma_C = \frac{\pi e^4 Z_a^2 Z_b^2}{E_{cm}^2} \ln \Lambda$$

where the parameter $\Lambda = 12\pi n \lambda_D^3$, n is the particle density and λ_D is the Debye shielding length. Λ accounts for shielding of long-range electrostatic forces in the plasma. The parameter $\ln \Lambda$ varies by less than a factor of 3 for plasmas of interest in fusion research[44] (Table 4).

The ratio of Coulomb scattering to fusion events is given by the ratio of their cross-sections, each plotted in Fig. 6. This ratio is an important dimensionless parameter of a fusion system. It is a function of temperature, but only a weak function of density: both Coulomb scattering and fusion rates are proportional to n^2 leading to a cancellation in their ratio, but the Coulomb logarithm introduces an additional weak n dependence into $\sigma_{C>}$

[44] F. F. Chen, *Introduction to Plasma Physics and Controlled Fusion* (1984), Ch. 5.6, p. 169.

Table 4. Coulomb shielding parameters for some fusion-relevant plasmas.

KT_e (eV)	n (m^{-3})	ln Λ	
0.2	10^{15}	9.1	(Q-machine)
2	10^{17}	10.2	(lab plasma)
100	10^{19}	13.7	(typical torus)
10^4	10^{21}	16.0	(fusion reactor)
10^3	10^{27}	6.8	(laser plasma)

3.2 Reaction Rates

For a DT reaction, the rate (fusions per unit volume per second) is $\Gamma = n_D n_T \langle \sigma v \rangle$, where n is the number density of reactants and <σv> is the velocity-weighted average of the cross section over the Maxwellian distribution of (relative) velocities. Thus, for a plasma at temperature T

$$\langle \sigma v \rangle \sim \int_0^\infty dE\, S(E) e^{-E/T}\, e^{-R/E^{1/2}}$$

where the integral is over E, the relative energy of the collision. If one neglects the energy dependence of $S(E)$, then the integrand peaks at $E^* \approx \left(\frac{RT}{2}\right)^{2/3}$ or $\frac{E^*}{T} \approx \left(\frac{R^2}{4T}\right)^{1/3}$. For the DT reaction, R = 34.37 keV$^{1/2}$, so at T = 10 keV, $E^*/T \approx 3.1$, implying that most reactions occur in the extreme tail of the Maxwellian distribution (*i.e.*, only the very fastest particles participate). Inclusion of $S(E)$ in the integrand shifts this result to $E^*/T = 3.5$. Representative reaction rates are shown in Fig. 7.

3.3 Thermodynamics

For the DT reaction, the energy released is ΔU = 17.6 MeV per reaction. This energy is equivalent to 3.4×10^8 MJ/kg of DT fuel. For comparison, burning of methane releases 55 MJ/kg fuel. The power density (power per unit volume) of a fusion reaction is $P = \mathrm{n_D n_T}\, \langle \sigma v \rangle\, \Delta U$. To achieve scientific break-even, the power released must exceed the power supplied to assemble the fuel. The latter can be estimated by U_{int}/τ_E, where U_{int} is the internal thermal energy in the plasma, and τ_E is the energy dissipation time. The internal energy is well described by a simple ideal gas model, with $\frac{3}{2}T$ of energy per particle (where T is measured in energy units). In a neutral plasma $n_{electron} = n_{ion}$, and if one assumes all species have the same temperature, then for the DT reaction, $U_{int} = 3\, T\, n_{ion}$.

Assuming a 1:1 mixture of D and T (so $n_D = n_T = \frac{1}{2} n_{ion}$), then the break-even condition becomes:

$$n\tau_E \geq \frac{12}{\Delta U} \frac{T}{\langle \sigma v \rangle}$$

In MCF systems where the maximum pressure is relatively constant (set by the maximum magnetic field), then it is customary to multiply both sides of the above expression by T, yielding:

$$nT\tau_E \geq \frac{12}{\Delta U} \frac{T^2}{\langle \sigma v \rangle}$$

By the ideal gas law, the quantity $n\,T$ is the pressure. The quantity $\frac{T^2}{\langle\sigma v\rangle}$ has a minimum around T = 14 keV, and substituting this value, leads to the Lawson criterion, $n\tau_E T \geq 3\ \times 10^{21}\ \mathrm{keV\ s/m^3}$. Fig. 7 shows that this expression is approximate, valid for DT only at T = 14 keV. Higher or lower temperatures require a higher value for the triple product.

At thermonuclear temperatures (T < 100 keV), DT has the largest reaction rate by two orders of magnitude and hence is the favored candidate for most approaches to fusion energy. The $p^{11}B$ reaction has been considered as an aneutronic alternative to DT. Despite the large charge product (Z = 5 for boron), its rate of energy production per ion ($\Delta U \times \langle\sigma v\rangle$) is within an order of magnitude of DT at temperatures above 150 keV (Fig. 7). A resonance in the compound ^{12}C system helps here, although the high temperatures are much more challenging than those required for DT. The presence of 5 electrons per B atom doubles the heat capacity of the $p^{11}B$ plasma relative to DT, assuming electrons and ions are at the same temperature. Thus twice as much energy must be delivered to $p^{11}B$ relative to DT to reach the same temperature.

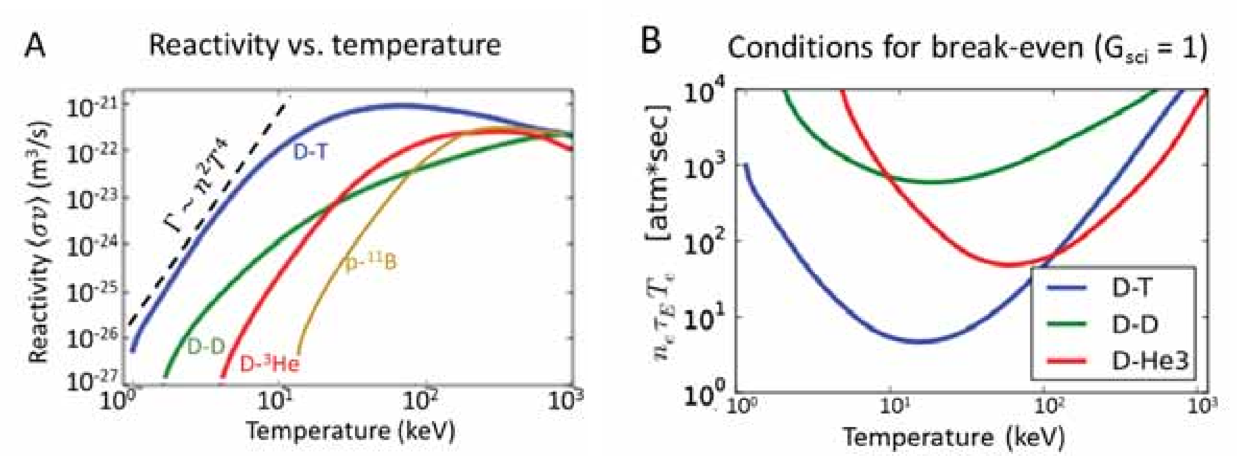

Figure 7. Thermodynamics of fusion reactions. A) Reactivity (reaction rate per particle) of the commonly studied fusion reactions. At temperatures < 100 keV, the DT reaction is most favorable, but at temperatures > 100 keV the $p^{11}B$ reaction becomes comparable. B) Triple product to achieve scientific break-even as a function of temperature. For the DT reaction, the optimal temperature is 14 keV.

3.4 Dissipation

Minimizing energy loss to the ambient environment is the primary challenge in every controlled fusion experiment. Energy is lost through photons, neutrons, and charged particles (electrons and ions). Neutrons escape freely and charged particle loss is described by the confinement time (which is different for the reactants than for the energetic charged products of the reactions).

Loss through photons is primarily via Bremsstrahlung radiation produced by electron-ion collisions. The radiative power dissipation is:

$$P_B \sim T_e^{1/2}\, n_e \sum_{ions} n_i Z_i^2 = \frac{Z_i^2 n_i n_e}{[7.69\ \times 10^{18} m^{-3}]^2} T_e[eV]^{\frac{1}{2}}$$

Figure 7A shows that for temperatures < 10 keV, the DT reactivity grows approximately as T^4, whereas the Bremsstrahlung power only grows as $T^{1/2}$. Both quantities scale as n^2, so their ratio is insensitive to density. For low-Z plasmas (e.g. DT), Bremsstrahlung radiation becomes insignificant relative to fusion power at temperatures above 4 keV. However, the Z^2 dependence imposes severe requirements on plasma purity. For instance, several MIF concepts contemplate using a Pb (Z = 82) liner to compress the plasma. Pb impurities at a concentration of ~10^{-4} relative to the fuel will approximately double the Bremsstrahlung rate.

While most analyses of MIF concepts have focused on minimizing transport of fuel nuclei to the reactor walls, **we note that transport of high-Z liner atoms into the fuel is a potentially serious problem that has received inadequate attention.**

For solid and liquid liners, the problem of liner-fuel mix can be divided into two parts: 1) ejection of atoms from the liner surface; and 2) transport of these ions through the magnetized fuel. Fig. 8 shows the equilibrium density of Pb atoms in the vapor phase as a function of the surface temperature of molten Pb (melting temperature 601 K). This vapor pressure places a lower bound on the Pb density in the vapor phase during a MIF experiment with a Pb liner. During fuel compression, the Pb surface may be heated by plasma-wall collisions and by Joule heating from induced electric currents; and Pb atoms may be sputtered off the surface by high-energy neutrons and ions escaping from the plasma. Characterization of these effects will require an experimental program to study liner-plasma interactions. The results of these experiments should be the joint distribution of velocities and ionization states of atoms exiting the liner surface when in contact with the plasma under realistic conditions.

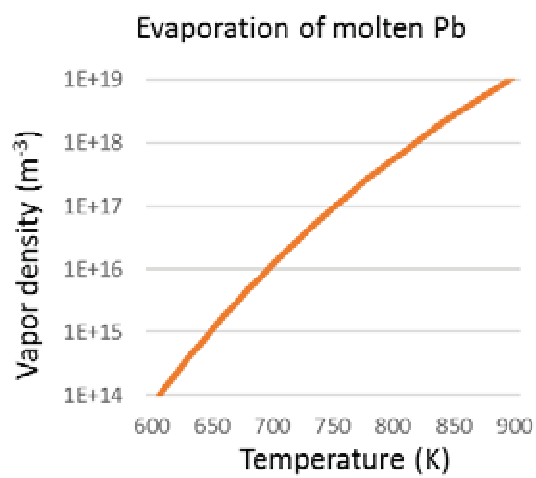

Figure 8. Equilibrium vapor pressure of lead as a function of temperature.

Liner-atom transport is also a complex process. Initially, neutral liner atoms will be able to penetrate the plasma without deflection by magnetic fields. As the evaporated or sputtered liner atoms heat, they will increase in both velocity and ionization state, while also affecting plasma parameters via Bremsstrahlung cooling. An improved understanding of this transport will be critical for mitigating technical risk in Pb liner-based approaches; and perhaps in other approaches with high-Z liners as well. This understanding will likely require a combination of experimental and computational work.

In addition to Bremsstrahlung radiative losses, high-Z impurities have another route to cool plasmas in pulsed systems: initially neutral liner atoms will absorb energy from the plasma as they heat and ionize. The n^th^ electron to be ionized from a liner atom will absorb $I_E^n + \frac{3}{2}k_B T$ of energy from the plasma, where I_E^n is the n^th^ ionization energy and $\frac{3}{2}k_B T$ reflects the translational kinetic energy after ionization.